



Throughput Optimization in Heterogeneous MIMO Networks: A GNN-based Approach

Ta-Yang Wang
University of Southern California
Los Angeles, USA
tayangwa@usc.edu

Hongkuan Zhou
University of Southern California
Los Angeles, USA
hongkuaz@usc.edu

Rajgopal Kannan
US Army Research Lab
Los Angeles, USA
rajgopal.kannan.civ@mail.mil

Ananthram Swami
US Army Research Lab
Adelphi, USA
ananthram.swami.civ@army.mil

Viktor Prasanna
University of Southern California
Los Angeles, USA
prasanna@usc.edu

ABSTRACT

With the development of 5G and IoT networks, Device-to-Device (D2D) communication has become a major paradigm in wireless communication. Most existing approaches for D2D resource allocation are usually time consuming and demand a high computational budget, especially in heterogeneous deployments where the D2D links have different configurations (i.e., different number of transmit and receive antennas). Recently, Graph neural networks (GNNs) have been proposed to solve many problems in the networking domain and have significantly outperformed traditional algorithms, including throughput optimization problems in D2D networks. However, existing throughput optimization works either only apply to MISO or SISO D2D networks or require extremely long runtime on MIMO D2D networks, which makes it hard to apply them in real-world D2D applications. In this paper, we consider the throughput prediction problem across a fixed association of transmitters and receivers to maximize the total throughput in heterogeneous MIMO D2D networks. We model the interference between different link types as heterogeneous edges and learn the optimal beamforming policy using a heterogeneous GNN. Simulation results show that our proposed GNN-based approach achieves a significant speedup compared with the state-of-the-art algorithm, while providing robust performance on large-scale synthetic datasets.

CCS CONCEPTS

• **Networks** → **Network performance analysis.**

KEYWORDS

beamforming, resource allocation, device-to-device communications, graph neural network, heterogeneous network

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1 INTRODUCTION

Effective resource allocation plays a crucial role for performance optimization in wireless networks. However, typical resource allocation problems are non-convex and computationally challenging [9, 10, 13]. This is because the utility functions under practical constraints are hard to optimize, especially when the number of mutually interfering links is large. Since multiple antenna technologies play a key role in communication systems, solving the beamforming and power control problem is crucial to achieving optimal performance in massive multiple-input multiple-output (MIMO) networks.

Great efforts have been put forward to approximate the classic weighted minimum mean square error (WMMSE) algorithm [12] for throughput optimization in terms of the weighted sum rate. Shen et al. [7] extend fractional programming (FP) theory with the goal of obtaining efficient suboptimal solutions to resource allocation problems in wireless communication networks. The iterative algorithm involves complicated computations such as matrix inversion in each iteration. Its high computational complexity makes it infeasible to be deployed to real-time applications.

Inspired by recent successes in machine learning (ML), researchers have been applying ML-based methods to solve NP-hard optimization problems in wireless networks [3, 5, 6]. In particular, traditional ML models such as MLP and CNN have been used to approximate the WMMSE algorithm. However, these approaches are not very effective at capturing complex network parameters such as topology and channel state information (CSI). A shortcoming of these ML models is the lack of scalability and generalization in large-scale resource allocation problems. Although they are able to achieve near-optimal performance for small-scale wireless networks, the performance of these methods drop drastically as the network size increases. This motivates the study of incorporating the structures of wireless networks into neural network architecture.

Graph neural networks (GNNs) have shown strong capability in exploiting non-Euclidean data, such as CSI. Therefore, GNN-based approaches have been proposed for resource allocation problems in D2D wireless networks [2, 4, 5, 10, 13]. However, these approaches are limited to scenarios involving either only MISO or only SISO links, limiting their applicability to real-world networks consisting

of a mixture of SISO, MISO and MIMO links. Table 1 summarizes the related work on the sum rate maximization problem. In the Link Type column, heterogeneous means that the environmental information (e.g., the number of transmit/receive antennas) may be diverse, homogeneous otherwise. The supported antenna system is indicated in parentheses.

We propose a GNN-based framework for the throughput optimization problem in heterogeneous MIMO networks. We treat the interference between different link types as heterogeneous edges, and then learn the optimal structure using heterogeneous GNN trained in an unsupervised manner.

The main contributions of this paper are:

- We extend the heterogeneous GNN-based framework for MISO links (HIGNN [13]) to MIMO links, which is a more general model in real-world scenarios.
- Experimental results show that our proposed model achieves significant speedup compared with FP and sum rate improvement over HIGNN.

The rest of this paper is organized as follows: Section 2 describes the system model as well as the formulation of the beamforming problem, and the graph representation for capturing the interference relations between distinct links. Section 3 introduces our GNN-based approach for the beamforming problem based on our graph representation. Section 4 provides numerical results to validate the performance of the proposed framework. Finally, Section 5 concludes the paper.

2 BACKGROUND

In this section, we formulate the throughput optimization beamforming problem in heterogeneous MIMO networks, and then describe how we model interference relations as a heterogeneous graph.

2.1 Notations

We use lower case to denote scalars, bold lower case to denote vectors, bold upper case to denote matrices, and Euler script to denote sets. We use \mathbb{R} to denote the set of real numbers, \mathbb{C} to denote the set of complex numbers, and \mathbf{I} to denote the identity matrix. For a matrix \mathbf{M} , we use \mathbf{M}^{-1} to denote its inverse and \mathbf{M}^\dagger to denote its matrix conjugate transpose.

2.2 Problem Definition

As illustrated in Fig 1, consider a heterogeneous MIMO network with a set of transmitters \mathcal{J} and a set of receivers \mathcal{I} , in which different links may have different features, e.g., varying number of transmit/receive antennas. We assume that only one data stream per link is supported. The throughput optimization problem considered here is to design transmit beamformers for the data stream in each active link.

Formally, suppose there are $T \times R$ types of links, denoted by $\mathcal{L} = \{\ell_{t,r} : 1 \leq t \leq T, 1 \leq r \leq R\}$, where the number of transmit antennas is N_t and the number of receive antennas is N_r for link type (t, r) . Let $\mathbf{H}_{i\ell, j}$ be the channel response from transmitter $j \in \mathcal{J}$ to receiver $i \in \mathcal{I}$ on link type ℓ . We assume that the channel state information (CSI) is completely known. Let σ^2 denote the variance of the additive white Gaussian noise (AWGN).

Introduce variable $\mathbf{x}_{i\ell} \in \mathbb{C}^{N_t}$ as the beamformer at transmitter i of type ℓ . The data rate to a receiver $j \in \mathcal{J}$ of type ℓ is computed as follows:

$$R_{i\ell}(\mathbf{X}) = \log \left(1 + \mathbf{x}_{i\ell}^\dagger \mathbf{H}_{i\ell, i}^\dagger \mathbf{S}_{i\ell}^{-1} \mathbf{H}_{i\ell, i} \mathbf{x}_{i\ell} \right), \quad (1)$$

where

$$\mathbf{S}_{i\ell} = \sigma^2 \mathbf{I} + \sum_{(j, \ell') \neq (i, \ell)} \mathbf{H}_{i\ell, j} \mathbf{x}_{j\ell'} \mathbf{x}_{j\ell'}^\dagger \mathbf{H}_{i\ell, j}^\dagger. \quad (2)$$

We use the weighted sum rate as the utility function in our optimal beamforming, where the weights account for fairness. Mathematically, given non-negative weights $w_{i\ell} \geq 0$ indicating the priority of link $i\ell$ and the power constraint P_{\max} , the throughput optimization problem can be formulated as

$$\begin{aligned} \max_{\mathbf{X}} \quad & \sum_{i, \ell} w_{i\ell} R_{i\ell}(\mathbf{X}) \\ \text{subject to} \quad & \|\mathbf{x}_{i\ell}\|_2^2 \leq P_{\max}, \forall i, \ell. \end{aligned} \quad (3)$$

The problem is nonconvex with vector variables and computationally challenging. In particular, on SISO networks, the beamforming design degrades to a power control problem.

2.3 Graph Representation

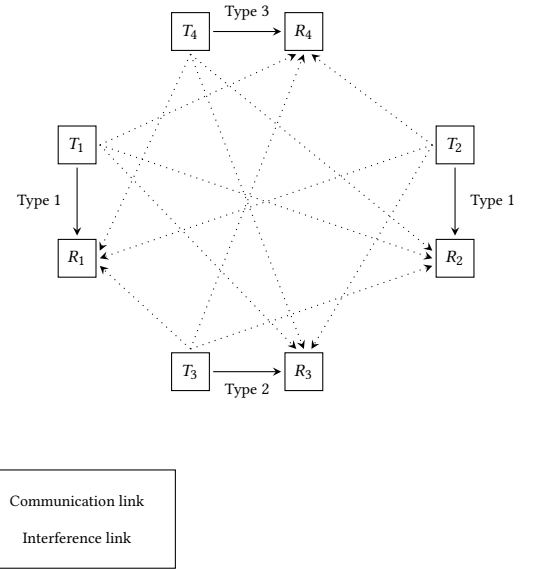


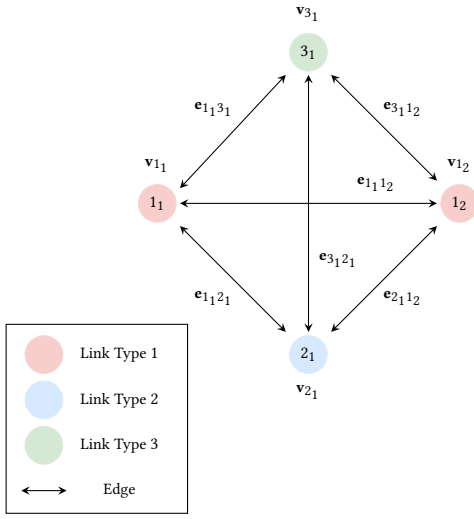
Figure 1: An example of a heterogeneous MIMO network with three types of links.

We describe how to model the interference relations between different types of links using heterogeneous graphs for throughput maximization in heterogeneous MIMO networks. The relational modeling is similar to that in [13].

The wireless network is modeled as a heterogeneous graph $G = (V, E, R)$, where each communication link is regarded as a node $v \in V$, each interference link is regarded as an edge $e \in E$, and relations $r \in R$ are adopted to identify node types associated with edges. Figure 2 illustrates the heterogeneous graph for the network

Table 1: List of the papers in the wireless network

Year	Problem	Link Type	Solution	Paper
2011	Beamforming	Heterogeneous (MIMO)	WMMSE	Shi et al. [12]
2018	Power Control and Beamforming	Heterogeneous (MIMO)	Fractional Programming	Shen et al. [7]
2019	Link Scheduling	Homogeneous (SISO)	CNN	Cui et al. [3]
2019	Power Control	Homogeneous (SISO)	GCN	Shen et al. [10]
2020	Power Allocation	Homogeneous (MISO)	REGNN	Eisen et al. [4]
2021	Link Scheduling	Homogeneous (SISO)	Graph Embedding	Lee et al. [5]
2021	Power Allocation	Homogeneous (SISO)	UWMMSE	Chowdhury et al. [2]
2021	Power Control and Beamforming	Heterogeneous (MISO)	HIGNN	Zhang et al. [13]
2021	Power Control and Beamforming	Homogeneous (MIMO)	UWMMSE	Chowdhury et al. [1]
2022	Power Control and Beamforming	Heterogeneous (MIMO)	GNN	Ours

**Figure 2: A heterogeneous graph modeling the interference relation in Fig. 1.**

in Figure 1. The node features of vertex i and edge features \mathbf{e}_{ij} of edge (i, j) depend on the channel gains between the two nodes of the corresponding communication/interference links.

Let $\mathcal{N}_i = \{j \in V \mid (j, i) \in E\}$ be the neighbor set of vertex i . Attributes of each vertex i and edge (i, j) are characterized by \mathbf{v}_i and \mathbf{e}_{ij} . Denote the i -th vertex of type ℓ by i_ℓ , whose incident neighbor nodes under relation $r = (\ell', \ell)$ is $\mathcal{N}_{i_\ell}^{(\ell')} = \{j \mid (j_\ell, i_\ell) \in E\}$. Node features are held by $\mathbf{V} = \{\mathbf{V}_\ell\}_\ell$, where ℓ specifies node types and $[\mathbf{V}_\ell]_i = \mathbf{v}_{i_\ell}$. Edge features are collected in $\mathbf{E} = \{\mathbf{E}_{\ell\ell'}\}_{\ell, \ell'}$, where $[\mathbf{E}_{\ell\ell'}]_{ij} = \mathbf{e}_{i_\ell j_{\ell'}}$ if edge $(i_\ell, j_{\ell'})$ exists and $\mathbf{0}$ otherwise.

In summary, we model each communication link as a vertex and each interference link as an edge. Attributes of each vertex include the weight and direct channel response. Edge features consist of the channel response from the interference links. Note that the size of the node/edge features may vary since there are different link types having distinct number of transmit and receive antennas.

3 METHODOLOGY

In this section, we develop an efficient GNN-based framework for general resource allocation problems in heterogeneous MIMO networks. We first introduce heterogeneous GNNs and then discuss the design of the learning framework for optimal beamforming.

3.1 Graph Neural Network

We introduce a GNN-based framework to solve the optimal beamforming in heterogeneous MIMO networks, by employing the heterogeneous GNN proposed in [13]. To find a policy f that maps the heterogeneous graph G built in Section 2.3 to estimate the beamforming vectors \mathbf{X} , we parameterize the policy by a learnable parameter θ as f_θ , the estimate of the beamforming vectors is $\hat{\mathbf{X}} = f_\theta(G)$.

GNNs iteratively update the representation of each node by aggregation and combination operations. The update rule is given as follows:

$$\alpha_v^{(k)} = \text{AGGREGATE}^{(k)} \left(\left\{ \beta_u^{(k-1)} : u \in \mathcal{N}(v) \right\} \right) \quad (4)$$

$$\beta_v^{(k)} = \text{COMBINE}^{(k)} \left(\beta_v^{(k-1)}, \alpha_v^{(k)} \right) \quad (5)$$

where $\alpha_v^{(k)}$ denotes the feature aggregated by node v from its neighbors at layer k . $\beta_v^{(k)}$ represents the feature vector of the node v at layer k .

The neighborhood aggregation is expected to capture the permutation invariance property of the interference channel, which can be achieved by a permutation-invariant operation (e.g., sum, mean, and maximum). Since dimensions of edge features change with antenna numbers, these features from different relations should be treated separately, as suggested in [13]. Therefore, we assign each relation $r = (\ell', \ell)$ individual update functions ϕ_r^v and ϕ_r^e , parametrized by multi-layer perceptrons (MLPs).

$$\mathbf{e}_{j_{\ell'} i_\ell}^{(k)} = \phi_{(\ell', \ell)}^e \left(\mathbf{v}_{j_{\ell'}}^{(k-1)}, \mathbf{e}_{j_{\ell'} i_\ell}^{(0)} \right), \quad (6)$$

$$\mathbf{v}_{\ell', i_\ell}^{(k)} = \phi_{(\ell', \ell)}^v \left(\mathbf{v}_{i_\ell}^{(k-1)}, \max_{j \in \mathcal{N}_{i_\ell}^{(\ell')}} \mathbf{e}_{j_{\ell'} i_\ell}^{(k)} \right). \quad (7)$$

Here initial edge attributes $\mathbf{e}_{j_{\ell'} i_\ell}^{(0)}$ are kept in all steps of edge update. This helps to stabilize training performance [13].

The per-relation updates $\{\mathbf{v}_{\ell',i_{\ell'}}^{(k)}\}_{\ell'}$ are merged to get the final vertex update $\mathbf{v}_{i_{\ell}}^{(k)}$ as

$$\mathbf{v}_{i_{\ell}}^{(k)} = \frac{1}{c_{i,\ell}} \sum_{\ell'=1}^{c_{i,\ell}} \mathbf{v}_{\ell',i_{\ell'}}^{(k)}, \quad (8)$$

where $c_{i,\ell}$ is the number of relations involved in updating $\mathbf{v}_{i_{\ell}}^{(k)}$.

During forward computation, each vertex takes its attributes as the initial input, and then a fixed number of updates are executed to obtain the beamforming vector. The power constraint is imposed by the activation function $\gamma(\mathbf{x}) = \frac{\sqrt{P_{\max}\mathbf{x}}}{\max\{\|\mathbf{x}\|_2, 1\}}$.

The loss function is the negative expectation of the weighted sum rate over different channel responses:

$$-\mathbb{E}_{\mathbf{H}} \left[\sum_{i,\ell} w_{i\ell} \log \left(1 + \mathbf{x}_{i\ell}^{\dagger} \mathbf{H}_{i\ell,i}^{\dagger} S_{i\ell,i}^{-1} \mathbf{H}_{i\ell,i} \mathbf{x}_{i\ell} \right) \right]. \quad (9)$$

Backpropagation of Eq. 9 is done by updating the learnable parameter $\boldsymbol{\theta}$ of the GNN in an unsupervised manner.

We identify the ability of a policy to be implemented on varying network topologies. Recall that \mathbf{X} is the collection of the beamformers, \mathbf{V} and \mathbf{E} are the node and edge features of the corresponding heterogeneous graph. Then the permutation invariance of utility function F implies that the following holds for any permutation operator π

$$F(\pi \circ \mathbf{X}, \pi \circ \mathbf{V}, \pi \circ \mathbf{E}) = F(\mathbf{X}, \mathbf{V}, \mathbf{E}).$$

The permutation equivariance of the policy f_{θ} by GNN can be formulated as

$$\pi \circ \mathbf{X} = f_{\theta}(\pi \circ \mathbf{V}, \pi \circ \mathbf{E}),$$

Therefore, the permutation of vertices doesn't affect the outputs of GNNs. Compared to traditional ML-based algorithms and other neural networks such as CNNs, these properties suggest that GNNs are able to generalize to heterogeneous networks of various sizes and distinct scenarios.

3.2 Complexity Analysis

Assume that the total number of links in the network is L . Let $N = \max\{N_t, N_r\}_{1 \leq t \leq T, 1 \leq r \leq R}$.

Greedy: The greedy algorithm makes decisions for each link sequentially. When deciding whether to schedule the i -th link, it compares the sum rate of all links that have been scheduled so far, with and without activating the new link. The re-computation of the interference costs $O(i)$ computations. The overall complexity of the greedy algorithm is $O(1 + \dots + L) = O(L^2)$.

GNN: Since the GNN-based models only require forward computation, the computational complexity is asymptotically the same as that of the Greedy: $O(L^2)$.

FP: As analyzed in [8, 9], the update step of FP has a per-iteration computational complexity of $O(N^4 L^2)$ and the matching step has a computational complexity of $O(L^2 \log L)$. This is due to the fact that in beamforming design, bisection search is performed in each iteration of FP. It is noted that the computational complexity of FP is sensitive to the number of transmit/receive antennas. It is asymptotically higher than those of the Greedy and GNN-based approaches.

The main advantage of our proposed GNN-based framework is that it is able to reduce the computational complexity while achieving near-optimal performance compared to FP.

4 EXPERIMENTS

In this section, we describe the simulation setting for dataset generation and provide model details, followed by experimental results and computational analysis. The implementation is based on HIGNN using deep graph library (DGL) with PyTorch backend [13].

4.1 Setup

We simulate a heterogeneous MIMO network where all links share the same bandwidth. All transmitters and receivers are uniformly distributed in a square area of length D . We consider 16 types of links: SISO links, $N_t \times 1$ MISO links, $1 \times N_r$ SIMO links, and $N_t \times N_r$ MIMO links, where $N_t, N_r \in [2, 4, 8]$. The communication range of each link is set between $2m$ and $50m$. We determine path loss and shadowing in large-scale fading using the scaled distance-dependent model in [11]. Channel response is computed by multiplying the square root of the large-scale fading component with the small-scale fading component, where the latter is simulated by i.i.d. zero-mean complex Gaussian variables with unit variance. The noise variance at receiver and transmit power budget are normalized to 1. The system parameters are summarized in Table 2. During the generation of channel instances for training, numbers of all 16 types of links are all set to 4. Since channel responses are complex matrices, as implemented in [13], we separately feed the real part and the imaginary part of the channel responses \mathbf{H} to NN modules after normalization.

Table 2: System Parameters

Parameter	Value
Square area of length, D	1000 m
Pairwise distance, $d_{\min} - d_{\max}$	2 - 50 m
Bandwidth, B	5MHz

4.1.1 Baselines. For performance evaluation of our GNN model, we compare with the following baseline approaches.

- HIGNN [13]: uses message passing neural network in a heterogeneous MISO network with two types of links that hold different features (i.e., SISO and MISO links).
- Greedy: sorts all the links according to the channel response and schedules the links one by one. A link is chosen to be active only if that increases the sum rate.
- Fractional Programming (FP) [7]: develops iteratively closed-form updates using fractional programming to optimize throughput.

4.2 Performance Comparison

We evaluate the performance of our GNN model as follows: we generate the synthetic dataset of multiple types of links under MIMO settings as described in Section 4.1. As a sanity check, we also conduct experiments under two scenarios: 1) SISO links only, and 2) SISO and MISO links. These scenarios are reduced from the

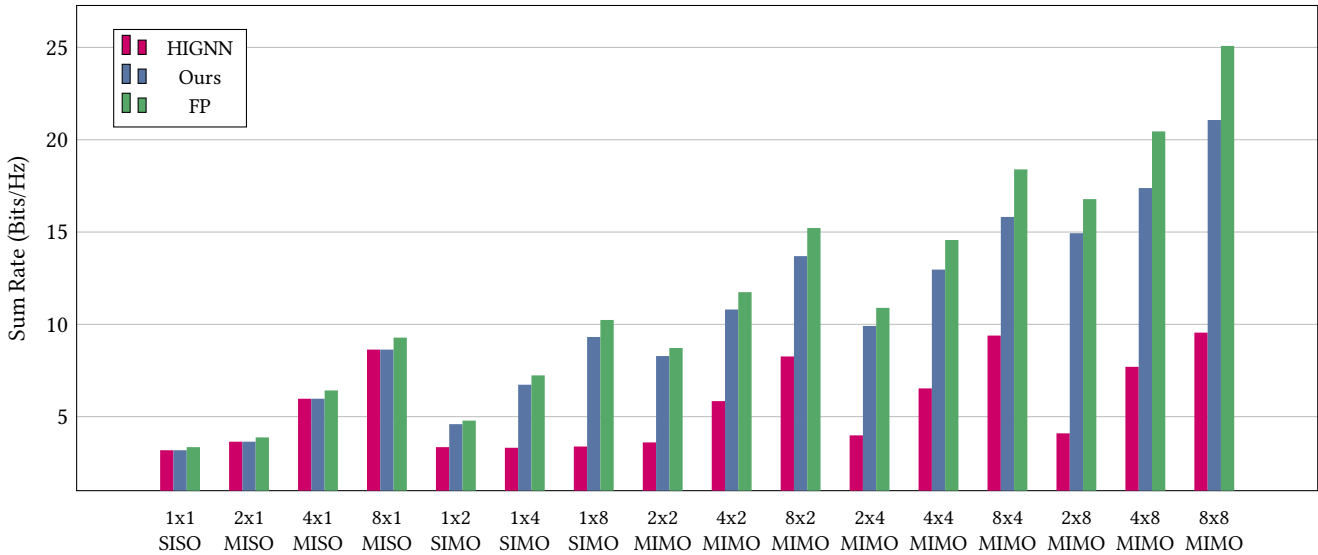


Figure 3: Sum Rate for heterogeneous MIMO networks (HIGNN: only SISO & MISO).

MIMO settings under the same deployment. For example, in the second scenario (SISO and MISO), all $N_t \times N_r$ MIMO links reduce to $N_t \times 1$ MISO links and all $1 \times N_r$ SIMO links reduce to SISO links. The simulation results are shown in Table 3. Note that we only test HIGNN and our proposed GNN-based framework under the most general antenna systems (i.e., a mixture of SISO & MISO links and all link types specified in Section 4.1, respectively).

It is seen that our proposed GNN-based approach is able to achieve an average of 88.9% of the weighted sum rate produced by the FP algorithm, while significantly outperforming the greedy algorithm in terms of the weighted sum rate. Therefore, our GNN-based approach is capable of handling interference relations in heterogeneous MIMO networks.

Table 3: Sum Rate (Bits/Hz) Under Different Scenarios

Scenario	Greedy	FP	HIGNN	Ours
SISO	19.83	46.93	-	-
SISO & MISO	23.87	87.40	81.82	-
ALL	35.92	186.65	-	165.79

Fig 3 illustrates the sum rate achieved by our GNN model and the benchmarks with respect to different antenna systems under the same deployment. Compared to HIGNN, our GNN model achieves similar performance on SISO and MISO links, with significant improvement on SIMO and MIMO links, especially as the number of antennas increases.

4.3 Running Time Performance

Compared to iterative algorithms, one advantage of GNN-based methods is the reduction in execution time. Therefore, we evaluate the running time performance of our GNN models and the FP algorithm to examine the computational complexity of these

approaches. As shown in [13], GNN-based methods implemented in an unsupervised manner are robust to the number of antennas.

We compare the running time for FP and GNN-based approaches under the system settings in Section 4.2, as shown in Table 4. Note that as in Section 4.2, we only test HIGNN and our proposed GNN-based framework under a mixture of SISO & MISO links and all link types specified in Section 4.1. The running time of FP grows significantly with the problem size, while those of the GNN-based approaches remain relatively small in magnitude. Therefore, our GNN-based framework is significantly faster than FP. This is because FP involves many iterations and has a much higher computational complexity (see Section 3.2). Based on the runtime results, both our GNN-based framework and HIGNN are fast and practical in real-world scenarios.

Table 4: Running Time (s) Under Different Scenarios

Scenario	FP	HIGNN	Ours
SISO	12.74	-	-
SISO & MISO	13.65	0.0357	-
All	34.37	-	0.0691

5 CONCLUSION

We developed an unsupervised learning-based framework for sum rate maximization in heterogeneous MIMO networks based on a heterogeneous GNN-based framework, which could capture interference in complex real-world wireless communication networks. For future work, we will investigate the possibility of designing a parameter sharing scheme for heterogeneous GNN, since there are multiple link types which share similar structure. This could significantly reduce the number of parameters and thus network complexity. We also plan to explore the optimal beamforming structures in a more general setting, for example, the flexible association

model in [9] that allows flexibility among multiple possible associations between transmitters and receivers.

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